

THINKING WITH AND THROUGH EXAMPLES

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In my talk I will discuss the roles that examples play (or could play) in mathematical thinking, learning, and teaching. I draw mainly on research I've been doing for over a decade that addresses this broad topic from several perspectives.

In terms of learning – I look at mathematical concepts (e.g., periodic function) and meta-concepts (e.g., definition) and examine the way interacting with examples may enhance understanding of these concepts. In terms of mathematical thinking, I look at mathematical proof and proving as a site for developing mathematical thinking (here I draw on my experience in designing and implementing an undergraduate course on Mathematical Proof and Proving (MPP), on my current work with Eric Knuth and Amy Ellis on the roles of examples in learning to prove, and my previous work with Uri Leron on generic proving, and with Orly Buchbinder on the roles of examples in determining the validity of mathematical statements). In terms of teaching, I try to unpack pedagogical considerations that teachers encounter when constructing or selecting instructional examples (this work I have done mainly with Iris Zodik), and try to characterize this kind of knowledge for teaching mathematics that appears to be crafted through experience.

I propose examining the field through three teaching/research settings that elicit example-use and example-based reasoning: Spontaneous example-use, evoked example-production, and example-provisioning (by the teacher or researcher).

WHY DEAL WITH EXAMPLES?

Over the past decade there has been an increasing interest among the community of mathematics education researchers in studying the roles, use, and affordances of examples in learning and teaching mathematics, at all levels. For example, at the 30th conference of PME in Prague, a Research Forum was devoted to exemplification in mathematics education (Bills, Dreyfus, Mason, Tsamir, Watson, & Zaslavsky, 2006). A special issue of *Educational Studies in Mathematics* (Vol. 69, No. 2) came out of that Research Forum and was published in 2008, followed by another special issue of *ZDM – Zentralblatt fuer Didaktik der Mathematik* on examples in mathematical thinking and learning (Antonini, Presemeg, Mariotti, & Zaslavsky, 2011).

The increasing attention to examples stems from the central role that examples play in learning and teaching, in general, and in mathematics and mathematical thinking, in particular. Examples constitute a fundamental part of a good explanation - a building block for good teaching (Leinhardt, 2001). According to Leinhardt (2001, p. 347), “For learning to occur, several examples are needed, not just one; the examples need to encapsulate a range of critical features; and examples need to be unpacked, with the

features that make them an example clearly identified.” Current work on mathematical exemplification indicates that using examples for explaining to oneself or to another person is a non-trivial challenge.

In this paper, the term *example* refers to a mathematical object for which one can answer the question: “What is this an example of?” In other words, the person who generates or selects it is able to articulate what property, principle, concept, or idea the specific example is a case of. Note that any example carries some attributes that are intended to be exemplified and others that are irrelevant. Skemp (1987) refers to the irrelevant features of an example as its ‘noise’, while Rissland (1991) suggests that “one can view an example as a set of facts or features viewed through a certain lens” (p. 190). When dealing with geometric concepts the notion of example has a unique nature (Zodik and Zaslavsky, 2007a). For example, there is no way to give an example of a ‘general’ triangle (except by a verbal description), since whatever triangle we sketch, it will always have salient features that are not general, as it cannot be both acute-angled and obtuse-angled. The above suggests that ‘examplehood’ is in the eyes of the beholder.

Examples are an integral part of mathematics and a critical element of expert knowledge (Rissland, 1978). In particular, examples are essential for concept formation, generalization, abstraction, analogical reasoning, and proof (e.g., Buchbinder and Zaslavsky, 2009; Dahlberg and Housman, 1997; Ellis, Lockwood, Dogan, Williams, and Knuth, 2013; Ellis, Lockwood, Williams, Dogan, and Knuth, 2012; Hazzan and Zazkis, 1999; Hershkowitz, 1990; Mason, 2011; Sandefur, Mason, Stylianides, and Watson, 2013), though there could be drawbacks in terms of use of examples, e.g., for proving (Iannone, Inglis, Mejía-Ramos, and Weber, 2011; Zaslavsky and Peled, 1997).

One way to examine how an individual understands a concept is by identifying elements of his or her concept image or example space. The collection of examples to which an individual has access at any moment, and the richness of interconnection between those examples, constitute his or her accessible example space (Bills et al., 2006). Example spaces are not just lists, but have internal structure in terms of how the elements in the space are interrelated. In my work, I consider an example space as the collection of examples one associates with a particular concept at a particular time or context. According to Mason and Goldenberg (2008), what determines the use of a concept is the example space one associates with it. This notion is closely related to Vinner and Tall’s idea of concept image (1981, 1983). Vinner and Tall use the term concept image to describe the total cognitive structure that is associated with a particular concept, which includes all the mental pictures and associated properties and processes. “It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures... Different stimuli can activate different parts of the concept image, developing them in a way which need not make a coherent whole.” (Tall & Vinner, 1981, p. 152). Example spaces are also dynamic and evolving. Thus, in orchestrating learning or conducting research it is important to identify

(limited) concept images and prototypical views of certain concepts, which the learners hold, and facilitate the expansion beyond “more of the same” examples.

Some parts of an example space may be more accessible at a given time than others (Mason and Goldenberg, 2008). The less accessible parts await an appropriate trigger to be used. Watson and Mason (2005) regard the notion of a personal example space as a tool for helping learners and teachers become more aware of the potential and limitations of experience with examples. In a group activity or discussion, an example suggested by one member may trigger access to a further class of examples for other members. When learners compare their examples, they often extend and enrich their example space. Moreover, once a connection is made it is strengthened and more likely to come to mind in the future (Mason and Goldenberg, 2008). Learners’ example spaces play a major role in what sense they can make of the tasks and activities in which they engage. Zaslavsky & Peled (1996), who coined the term *example space*, point to the possible effects of limited example spaces that (practicing and prospective) teachers hold with respect to a binary operation on their ability to generate examples of binary operations that are commutative but not associative or vice versa.

Research on exemplification in mathematics education has become very broad and includes different approaches to studying it, as well as competing and even seemingly contradicting findings. For example, Dahlberg and Housman (1997) indicate that students who generated examples of an unfamiliar concept more widely and visually gained a better understanding of the concept and were able to solve subsequent related problems better. On the other hand, based on their findings, Ianonne and her colleagues (2011) warn that simply asking students to generate examples is not necessarily productive for proving. In both studies the students were explicitly asked to use examples. Sandefur et al. (2013) argue that in order to understand the value of exemplification one needs to examine cases of spontaneous use of examples. They build on studies that suggest that example generation is beneficial for problem solving and proving when the participants generate and use examples spontaneously (Alcock and Inglis, 2008; Watson and Chick, 2011). In contrast to the two settings that foster learners’ generation of examples – either spontaneously or deliberately, there are also studies that examine instructional examples that are provided by a teacher (or researcher) with a certain goal in mind (e.g., Hershkowitz, 1987, 1990; Zodik and Zaslavsky, 2008). The intentions of such studies are twofold: to unpack design principles, and to examine the affordances that such examples or sequence of examples create, in terms of learning. These settings can be actual teaching sessions or research sessions (e.g., individual or group interviews, teaching experiments).

Whatever the learning environment (or research setting) involves – spontaneous generation of examples, deliberately evoked production of examples, and provisioning of examples, there is still a web of other factors that play a role, such as the mathematical topic and complexity of the focal concept or problem, prior experiences and knowledge (Alcock & Inglis, 2008), the kind of interactions that are facilitated (between the teacher/interviewer and the learner, and /or between learners in a small or

large group). For this matter, learners can be students at all levels as well as teachers who are engaged in these types of activities.

In my presentation I will use examples from my own work and work with colleagues of mine to illustrate what example generation and use can look like in these three different settings, and what possible affordances and limitations are entailed in each setting, acknowledging that each example has its specifics (in Skemp's terms – “noise”). In this paper, I elaborate on one example of a process of an evoked example -production, and mention briefly other cases (on which I will elaborate further in my presentation). These examples are related mostly to concept formation and proving. They serve as ‘existence proofs’ of exemplification in the service of learning mathematics and learning to teach mathematics. Included in learning to teach mathematics is using example generation as a diagnostic tool to identify students’ strengths and weaknesses (Zaslavsky & Zodik, in press; Zazkis & Leikin, 2007).

I turn to the three settings mentioned above.

SPONTANEOUS EXAMPLE-USE

Spontaneous-use of examples does not occur automatically. There are conditions that foster this type of behavior. For this to occur, special attention needs to be given to the nature of the task in which students engage. Typically, tasks that create a sense of uncertainty (Zaslavsky, 2005; Zaslavsky, Nickerson, Stylianides, Kidron, and Winicki-Landman, 2012) have the potential of raising the need to try out examples spontaneously. The uncertainty can be about a conjecture (i.e., whether it is true or false), or about a problem-solving situation for which the solver has no readily solution strategy that works (problems that require proving are included in this type). The examples can be generated randomly, just to get a sense if the conjecture holds for all cases, with the goal of building an intuition for whether or how to prove or disprove the conjecture. The examples may also be carefully selected and represented in a way that provides structure and potentially shed light on the main ideas of a proof, such as in generic proving (Leron and Zaslavsky, 2013; Malek and Movshovitz-Hadar, 2011; Rowland, 2001).

This type of setting allows studying what may come naturally to learners and experts, and what productive (or problematic) uses of examples can be anticipated. Moreover, the intention in studying spontaneous example-use is often aimed at building on this in other contexts or with other learners, for example, by explicitly introducing such forms of example use.

In a study on the roles of examples in learning to prove¹, we identified several manifestations of students’ productive use of examples, without prior direct instruction on uses of examples for proving. In particular, in a task-based individual interview

¹ Examples Project (NSF grant DRL-1220623, Eric Knuth, Amy Ellis, & Orit Zaslavsky, principal investigators).

some students used examples generically, and were able to build on a generic example to form a deductive argument that supports their assertion. One task was about the following conjecture: “When you add any consecutive numbers together, the sum will always be a multiple of however many numbers you added up”.

One student chose an example of 5 consecutive numbers and wrote: $5+6+7+8+9$. Then he represented the sum in the following way: $(7-2)+(7-1)+7+(7+1)+(7+2)$. The latter representation allowed him to see this example as a general case for any 5 consecutive numbers, and even for any odd number of consecutive numbers (since there will always be a middle term).

Zaslavsky and Shir (2005) studied students’ conceptions of a meta-concept: a mathematical definition. They did it through task based group sessions, in which the researcher did not interfere at all in the students’ discussions. The tasks required examining several possible definitions of a mathematical concept and determining whether each one is acceptable as a definition of the given concept. Interestingly, this task elicited rich example-based reasoning that led to shifts in students’ conception of a definition.

Note that although the settings described here elicit spontaneous (and often productive and desirable) example-use, it does not occur incidentally. It relies on careful choice and design of appropriate tasks.

EVOKED EXAMPLE-PRODUCTION

There are numerous manifestations of the value of explicitly asking students to generate examples of a concept or use examples for problem solving or proving (Watson & Mason, 2002, 2005; Hazzan & Zazkis, 1999; Zaslavsky & Zodik, in press). In terms of concept formation, this requirement may serve to get an idea of a person’s concept image (or example space) and motivate towards its expansion.

I turn to a group activity at a professional development workshop for in-service secondary school teachers, that explicitly required example-generation of a particular mathematical concept – a periodic function, followed by example-verification (for more detail see Zaslavsky and Zodik, in press). We (ibid.) used the generic task of: ‘Give an example of..., and another one..., and now another one, different from the previous ones...’. This type of task has been discussed in the literature (e.g., Hazzan & Zazkis, 1999; Watson & Shipman, 2008; Mason & Goldenberg, 2008; Zaslavsky, 1995). This activity calls for generation and verification of examples. The assumption is that the learners know the definition of the concept.

We chose the concept of a periodic function, in order to examine the current concept images teachers held, and motivate them to expand their example spaces associated with periodicity (Shama, 1998). Van Dormolen and Zaslavsky (2003) discuss the meta-concept of a mathematical definition, and illustrate it with the notion of a periodic function. They suggest the following pseudo-definition (p. 92, ibid) that is useful pedagogically, as it conveys the essence of the notion of a periodic function:

A pseudo-definition of a periodic function

A periodic function is a function that can be constructed in the following way: Divide the x-axis into equal-length segments, such as for example..., $[-39, -26]$, $[-26, -13]$, $[-13, 0]$, $[0, 13]$, $[13, 26]$, $[26, 39]$, ... Take any of these segments, no matter which one, and define a function on it, no matter how (e.g., as in Fig. 1).



Fig. 1

Then define another function on the whole x-axis, such that on each segment it behaves in the same way as the first function (as in Fig. 2).

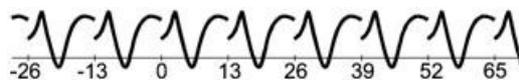


Fig. 2

Then the new function is a periodic function. Its values are repeated regularly.

Van Dormolen & Zaslavsky, 2003, p. 92

Inspired by this work, it appeared that the notion of a periodic function would lend itself well to generating and verifying examples with the goal of expanding the participants' example spaces, or concept images of a periodic function; it was anticipated that the teachers would mainly think of trigonometric functions as examples of periodic functions, thus, there would be many learning opportunities to expand their example spaces. In particular, we anticipated that the idea of constructing an example of a periodic function without knowing, or even being able to know, its analytic representation, basically similar to the above 'copying' approach of van Dormolen and Zaslavsky (2003), would be a new way of thinking of a periodic function.

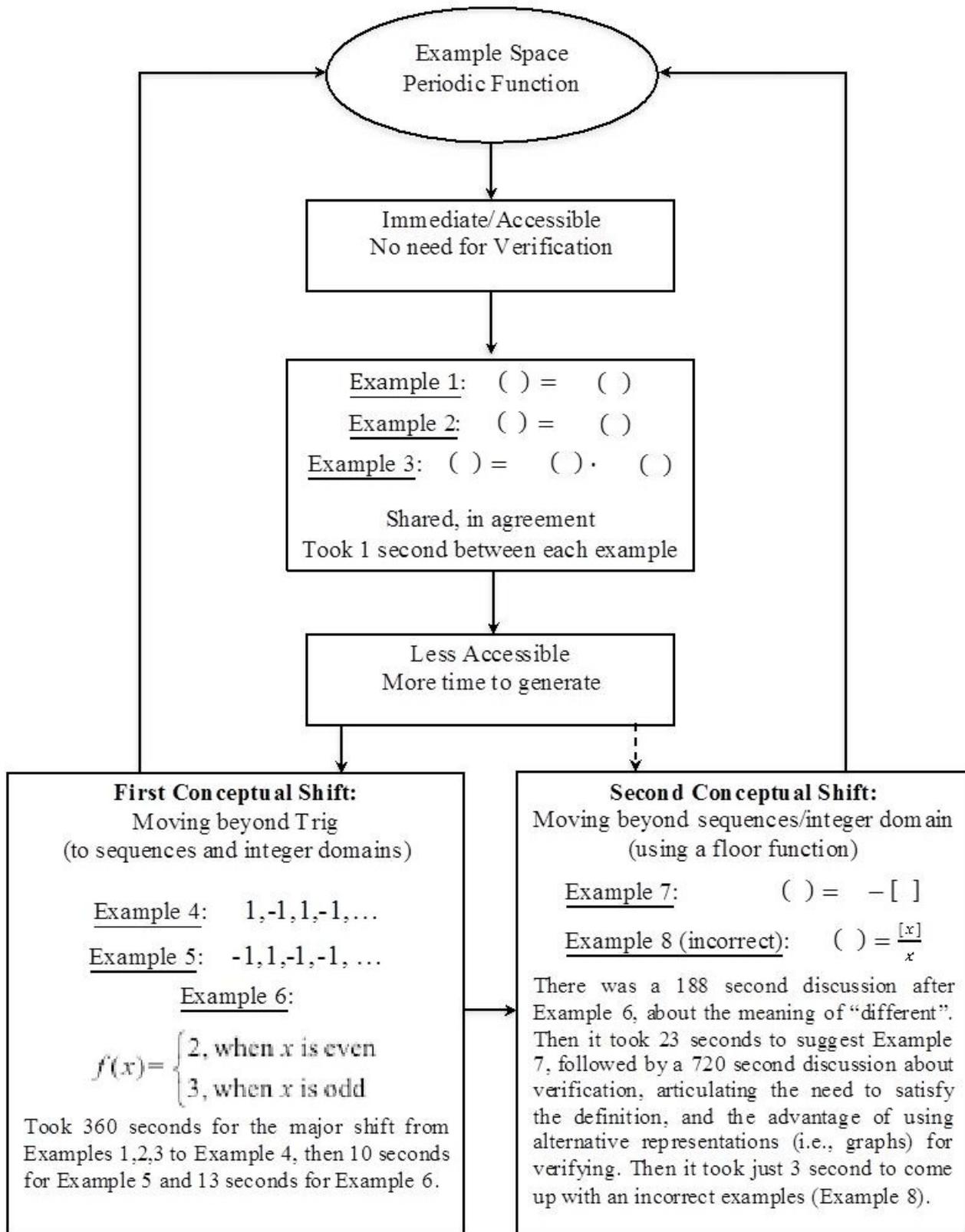


Figure 1: Collaborative expansion of an example space of a Periodic Function by evoked example-production

Figure 1 conveys the turning points in how the participants viewed a periodic function. After giving 3 familiar, rather prototypical and highly accessible examples (examples 1, 2, & 3), all drawn from the domain of trigonometric functions, they ran out of examples. This led to a discussion, in which one of the participants, Reli, suggested moving from the domain of trigonometric functions to special kinds of sequences. Based on Reli's idea, the group members helped her construct a specific example. This learning event triggered the first shift from regarding periodic functions as mostly (or even solely) combinations of trigonometric functions, to non-trigonometric functions. Example 4 can be seen as a conceptual shift in participants' views of a periodic function. It opened their eyes to similar cases and led to the construction of examples 5 and 6, by Hassan and Mary, respectively. These examples triggered the next discussion, as several participants, including Hanna, questioned the extent to which examples 4, 5 and 6 are essentially different. As a result, a more sophisticated example emerged (example 7), involving a floor function (by Hassan). This example led to a long discussion including group work surrounding ways to verify that example 7 is indeed a periodic function. Some verified this based on the symbolic representation of the function and some used its graphical representation. Interestingly, while periodicity lends itself naturally to visual representations, this did not come up spontaneously. However, at a certain point, participants suggested also approaching examples of a periodic function graphically (in the spirit of van Dormolen and Zaslavsky, 2003).

For examples 7 and 8, it was not easy to check whether they satisfy the definition of a periodic function. For that, several participants moved to a graphical representation, such as the one in Figure 2.

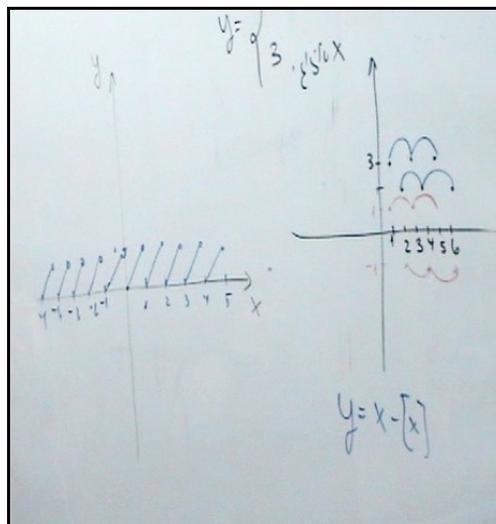


Figure 2: A graph of Example 7 that a participant drew on the board

For example 8 the graphical representation was difficult to draw, so it remained indecisive whether it was a periodic function or not. Only at a later stage they were able to find its graph (see Figure 3) and realize that it was not a periodic function.

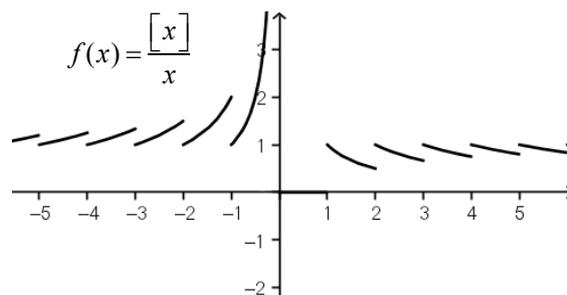


Figure 3: The graph of Example 8

The case of the periodic function represents an example-generation and verification eliciting learning environment that is characterized by an open-ended generic task of “give an example of...”. This learner-centered environment is characterized by the ongoing activity of generating examples of a given concept followed by the naturally evolving need to verify that the proposed examples satisfy the definition of the given concept or other sufficient conditions of the concept. The teacher’s or researcher’s main roles are choosing the focal mathematics concept, and orchestrating the discussions; it is critical that the teacher persists and pushes the learners to continue generating more and more examples that are different than the previous ones. As we see in Figure 1, the learning occurs once we go beyond the familiar and the accessible.

PROVISIONING OF EXAMPLES

Studies on how people learn from worked-out examples point to the contribution of multiple examples, with varying formats (Atkinson, Derry, Renkl, & Wortham, 2000). Such examples support the appreciation of deep structures instead of excessive attention to surface features. Other studies dealing with concept formation highlight the role of carefully selected and sequenced examples and non-examples in supporting the distinction between critical and non-critical features and the construction of rich concept images and example spaces (e.g., Hershkowitz & Vinner, 1983; Vinner, 1983; Petty & Jansson 1987; Watson & Mason 2005; Zaslavsky & Peled, 1996). In these studies, it is the role of the teacher or researcher to select and provide the specific examples that the learners will encounter. The choice of examples then is related to the learning or research goals.

The choice of an example for teaching is often a trade off between one limitation and another. Choosing examples for teaching mathematics entails many complex and even competing considerations, some of which can be made in advance, and others only come up during the actual teaching (Zaslavsky, 2010; Zodik & Zaslavsky, 2007b, 2008, 2009). The specific choice and treatment of examples are critical as they may shape students’ understandings by facilitating or hindering learning (Zaslavsky & Zodik, 2007; Rowland, Thwaites, & Huckstep, 2003).

The knowledge teachers need for meeting the challenge of judiciously constructing and selecting mathematical examples is a special kind of knowledge. It can be seen as core knowledge needed for teaching mathematics. In Ball, Thames, and Phelps’ (2008)

terms, it encompasses knowledge of content and students and knowledge of content and teaching, as well as “pure” content knowledge unique to the work of teaching. Teachers’ treatment of examples may reflect their knowledge base (Zaslavsky, Harel, & Manaster, 2006). Moreover, engaging teachers in generating or choosing instructional examples can be a driving force for enhancing these elements of their knowledge (Zodik & Zaslavsky, 2009). Teachers’ use of examples often leads to learning opportunities for themselves through which they gain pedagogical and/or mathematical insights (Zodik & Zaslavsky, 2009).

In examining the quality of instructional examples there are two main attributes that appear to make an example pedagogically useful, according to Bills et al. (2006). First, an example should be 'transparent' to the learner, that is, it should make it relatively easy to direct the learner’s attention to the features that make it exemplary. It should also foster generalization, that is, it should highlight the critical features of an example of the illustrated case, and at the same time point to its arbitrary and changeable features.

This notion of transparency is consistent with Mason and Pimm's (1984) notion of generic examples that are transparent to the general case, allowing one to see the general through the particular, and with Peled and Zaslavsky’s (1997) discussion of the explanatory nature of examples. Examples with some or all of these qualities have the potential to serve as a reference or model example (Rissland, 1978), with which one can reason in other related situations, and can be helpful in clarifying and resolving mathematical subtleties.

Provisioning of examples calls for special attention to how the learner interprets the example and what the learner notices (or fails to notice). The following example, taken from Zodik and Zaslavsky (2007b), illustrates this point. In a geometry lesson introducing the concept of a median of a triangle, the teacher used the following example (Figure 4) to illustrate a median:

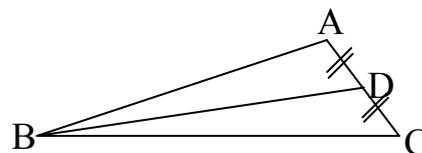


Figure 4: An initial example of a Median (BD)

Based on this example, a student suggested that any median is also an angle bisector. Following the student’s remark, the teacher modified the original example and presented the following case (Figure 5).

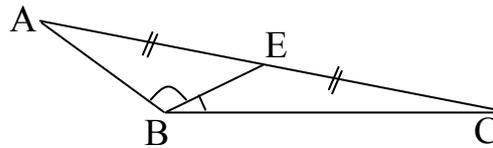


Figure 5: A revised example of a Median (BE)

Apparently, there are special entailments in visual/geometric examples. Basically, there is much ambiguity with respect to what visual information one is allowed to attend to and infer from a drawing and what not. Yet, the possible “mismatch” between a teacher’s intention and what students attend to is not restricted to visual examples.

CONCLUDING REMARKS

The three settings – spontaneous example-use, evoked example-production, and provisioning of examples are interrelated. Often in research we first want to examine what students do spontaneously and only at a later stage direct them to example use. There are times when we evoke students to generate their own examples, and then present them with additional examples that were either “missing” with respect to a goal we have in mind, or that can help shed light on their thinking.

In my talk I will elaborate on the affordances and limitations of each setting, draw connections and point to similarities across these settings.

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