

# *ELLs and Mathematics*

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## Factors Affecting ELLs' Success in Mathematics

### Limited prior knowledge and/or background knowledge

The impact of schooling difficulties may begin before attending school in the United States. Depending on the country of origin, students may not have had adequate education and/or may not have made adequate progress in school. The end result is that some ELLs may lack basic math skills and the ability to grasp the new concepts taught in required secondary school math courses.

Misconceptions of students' math skill levels, which are based upon their cultural background and upbringing, do not help the ELL student. For example, the myth that an Asian student is going to excel in math based upon race is a false understanding of the student's profile. The reality is that Asian students who lived in urban areas may not have attended school regularly and may not have the basic math skills that they are assumed to possess. Having had limited prior schooling, the ELL students who have had limited prior school may not have the basic computation skills required to succeed even in the first year of high school algebra or geometry.

### Cultural differences

Math is often considered to be a universal language where numbers connect people regardless of culture, religion, age or gender.

However, learning styles differ greatly in Eastern countries. In many Asian countries, rote memorization and self-study form the basis of schooling and learning. Thus, students may have little or no experience working in cooperative groups, let alone sharing and discussing how to solve problems.

Teachers must also be aware that some symbols serve different functions in different cultures. Use of the comma and decimal point varies from culture to culture. Students from South America, Asia, and many European countries use the comma in expressing currency values, whereas Americans use a period.

Some mathematical concepts may also differ in various countries, thus making it challenging for ELL students to re-learn math concepts. One example is measurement. Most countries around the world such as China, India, and France, use the metric system in weights and measures; only the United States, Liberia and Myanmar do not use the metric system. Imagine the mistake a student might make in assessing height in solving a math problem. The response given may be 1.52 meters, while the answer we in the United States are looking for is 5 feet tall.

Temperature is recorded in Celsius degrees in the metric system, although in the United States we use Fahrenheit when referring to the temperature. Think about how difficult it is to know that when the weather reports it is 46°F it is also 7°C.

These varying concepts in culture would not only affect the ELL students learning math, it may also impede their understanding of the material being taught. Early on in the class, teachers need to survey their students and learn their backgrounds in order to effectively address their needs.

## Linguistics

There is a big difference between everyday language, known as Basic Interpersonal Communication Skills, (BICS) and academic language, known as Cognitive Academic Language Proficiency (CALP) acquisition for ELLs (Cummins, 1979).

Many ELL students do not have a knowledge of academic English, the language used in school, thus making it even more difficult as they face the challenge of learning and using content-specific vocabulary.

Some linguistic challenges in math learning include:

- Learning *mathematics vocabulary*. Many of the words in mathematics are difficult to decode and learning to decode a technical language is important to learning math. Math vocabulary include words specific to mathematics, such as *equation, algebraic, etc.*, as well as everyday vocabulary that has different meanings when used in mathematical contexts. (See polysemous words below.) However, some mathematical terms may not translate well thus leaving the student with the task of really understanding the content taught.
- Students must learn to *associate mathematical symbols with concepts and the language used to express those concepts*. Example: the symbol / expresses the idea of something 'divided by'.
- Mathematical texts frequently use the *passive voice*, a complex and difficult structure for many non-English speakers. For example: *ten (is) divided by two and when 15 is added to a number, the result is 21; find the number*.
- Mathematics also uses *strings of words* to create complex phrases with specific meanings, such as *a measure of central tendency* and *square root*.

The complexities of reading math texts and understanding the language of math are real for all students. For the ELL student, these challenges can be frustrating and monumental. The effective math teacher needs to be aware of these factors that may hinder learning and address them in lessons.

Examples of Language Difficulties

## Polysemous words may be troublesome in the Math class.

Polysemous words, which are words with the same spelling and pronunciation but different meanings, can be confusing for the ELL student to understand.

Many words are used in math textbooks and teaching which differ from their everyday life meanings. The instruction of specific vocabulary is crucial because vocabulary knowledge correlates with math reading comprehension. Also, note that words functioning as a verb, a noun, or an adjective may also have different definitions.\*

Examples of polysemous words:

Word	Meaning in Everyday Life	Meaning in Math
angle	a viewpoint or standpoint	In geometry, it's the space within two lines.
mean	(adj) offensive* (v) to intend*	An average
table	furniture	An arrangement of numbers, symbols or words to exhibit facts or relations
volume	loudness	Amount, total of
tree	a plant	Tree diagrams
area	a space or surface	The quantitative measure of a plane or curved surface
root	the underground part of a plant	The quantity raised to the power $1/r$
gross	offensive, disgusting	The total income from sales
operation	medical surgery	A math process, addition, multiplication...
domain	territory	The set of values assigned
degree	diploma	The sum of the exponents of the variables in a algebraic term
expression	a look indicating a feeling	A symbol representing a value
order	a command.	In algebra, the degree
power	the ability to do something, strength	the product obtained by multiplying a quantity by itself one or more times (3 diff meanings)
Odd	bizarre	leaving a remainder of 1 when divided by 2. Numbers such as 3, 5...
even	smooth, straight	a number divisible by two

Examples:

- The dog wore a quizzical *expression* when Andy asked him to solve the math *expression*.

- Everyone *threw* Samantha mean looks when she reminded the teacher to include finding the *mean* in word problems for the math test.
- The beauty store *grossed* a total amount of \$1,700 for the day even though clients were *grossed out* by the smell of raw sewage permeating throughout the store.

For more words, visit the following website: <http://www.mathwords.com/>

## Syntactic features of word problems

The arrangement of words in a sentence, or syntax, plays a major role in understanding phrases, clauses or sentences. Faulty syntax is especially detrimental in the reading, understanding, and solving of word problems in math.

Here is an example of an algebraic expression, which would cause problems if translated word for word:

*The number "a" is 5 less than the number "b".*

In the example, the syntactic mistake would be in reading the sentence word for word as it is logical to do. Hence the student would undoubtedly write  $a = 5 - b$ . However, the sentence calls for the student to understand what? The correct answer would be

$$a = b - 5$$

## Semantic Features that May Cause Challenges for ELL Students.

*Synonyms:* add, plus, combine, sum

*Homophones:* sum, some; whole, hole

*Difficult expressions:* If...then, given that...

*Prepositions (phrasal verbs):* divided into vs. divided by, above, over, from, near, to, until, toward, beside

*Comparative constructions:* If Amy is taller than Peter, and Peter is taller than Scott, then Amy must be taller than Scott.

*Passive structures:* Five books were purchased by John.

*Conditional clauses:* Assuming X is true, then Y

*Language function words:* to give instructions, to explain, to make requests- What?

## Text Analysis

Word problems in math often pose a challenge because they require that students read and comprehend the text of the problem, identify the question that needs to be answered, and, finally, create and solve a numerical equation. Many ELLs may have difficulty reading and understanding the written content in a word problem.

### WORD PROBLEM 1

*In three more years, Miguel's grandfather will be six times as old as Miguel was last year. When Miguel's present age is added to his grandfather's present age, the total is 68. How old is each one now?*

#### Challenges:

1. This word problem may pose difficulty for the ELL student in distinguishing tenses as they relate to the word problem.

**The solution?** It may be useful for the teacher to draw a time line, have the student highlight all tense words in the word problem and determine which tense they are. See below:

In three **more years**, Miguel's grandfather **will be** six times as old as Miguel **was last year**. When Miguel's **present age** is added to his grandfather's **present age**, the total is 68. How old is each one **now**?

Past tense (-)	Present tense (your task to find out)	Future tense (+)
Was last year	present age now	more years will be

2. The same problem may pose another difficulty because understanding a concept is harder when the concept is made up of the relationship between two words.

**Solution?** The teacher might point out and create a chart of difficult words for students to be aware of when they are solving word problems. Identifying these words will help students understand that there is a relationship between two things (most often a comparison).

Miguel's grandfather will be six times **as old as** Miguel

Other difficult words/sentences taken from word problems include:

Her mother earns 5 times **as much as** her father.

I am 6 years **older** than you.

7 **divided by** 2 is.

Two numbers, **the sum of which** is 10.

Two numbers, **whose product is** 1.

### WORD PROBLEM 2:

*Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, you decide to mix 10% solution with 30% solution, to make your own 15% solution. You need 10 liters of the 15% acid solution. How many liters of the 10% solution and 30% solution should you use?*

### Challenges:

1. Unknown or confusing words for the ELL student.

**Solution?** The teacher could first look at the text and highlight key words which may cause problems for ELL students and review them.

Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, you decide to mix 10% solution with 30% solution, to make your own 15% solution. You need 10 liters of the 15% acid solution. How many liters of the 10% solution and 30% solution should you use?

2. Multiple meaning words highlighted in blue.

In lengthy word problems, ELL students will come across words with multiple meanings. This may be confusing which will only make understanding the text difficult. As the ELL students check the dictionary for these words, they will find it is necessary to use contextual clues to help them understand the text.

**Solution?** Again, highlight such words. Then, help students find contextual clues in the paragraph. This will clear up the correct meaning of the word. Words such as *mix* help indicate that the word, *solution* in this word problem does not mean 'the answer' as it is often understood in math vocabulary. Rather, in this sentence, the word *solution* in the context of a lab is a liquid to be mixed with another liquid.

Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, you decide to mix 10% solution with 30% solution, to make your own 15% solution. You need 10 liters of the 15% acid solution. How many liters of the 10% solution and 30% solution should you use?

Other words with multiple meanings in this word problem include *ships* and *test*.

**Ships-** "...your supplier only *ships* a 10% solution." In this sentence, *ships* is a verb, meaning to send. Students may know only the more commonly used meaning which is the noun- a vessel that goes into the water.

### Effective Strategies for Teaching Mathematics

The need to develop new strategies for language instruction in the math classroom is great simply because the population of students in our schools come from varying language backgrounds. Following are some effective strategies to teach math.

#### STRATEGY: *Venn Diagram for Comparison-Contrast*

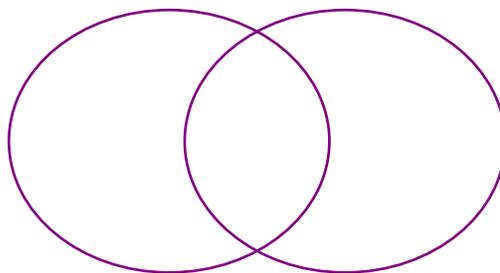
##### Description

A **Venn Diagram** is a drawing in which circles represent groups of items sharing common properties. The common characteristics are written in the space where the two circles overlap. The different characteristics are written on the inside of the separate parts of each circle.

##### Application of Strategy

Use the Venn diagram by asking students to compare and contrast shapes, numbers and concepts.

Example 1:

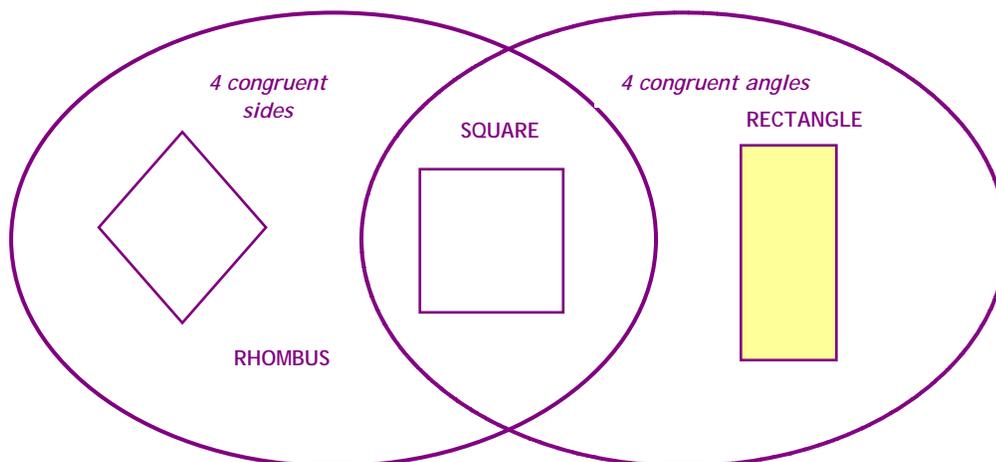


*For a perfect venn diagram organizer, go to this website:  
[http://www.readwritethink.org/lesson\\_images/lesson378/venn.pdf](http://www.readwritethink.org/lesson_images/lesson378/venn.pdf)*

Use the Venn diagram to show the relationships between shapes.

Facts Given:

*A rhombus has four congruent sides. A rectangle has four congruent angles. A square has four congruent sides and four congruent angles.*



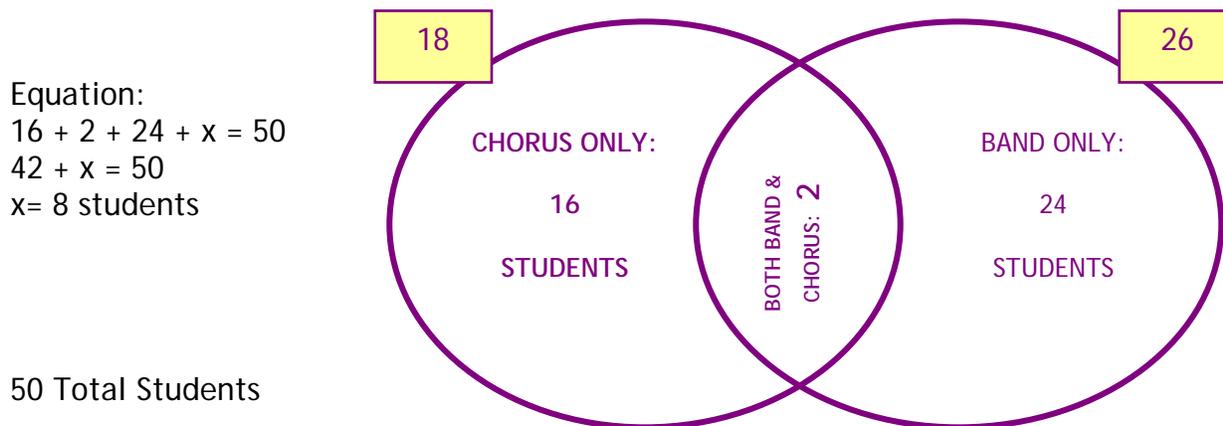
Example 2:

Use Venn Diagrams to solve word problems.

Word Problem:

*In a class of 50 students, 18 take Chorus, 26 take Band and 2 take both Chorus and Band. How many students in the class are not enrolled in either Chorus or Band?*

Task: Use the Venn Diagram to show the relationship of the Band and the Chorus.



Equation:  
 $16 + 2 + 24 + x = 50$   
 $42 + x = 50$   
 $x = 8$  students

50 Total Students

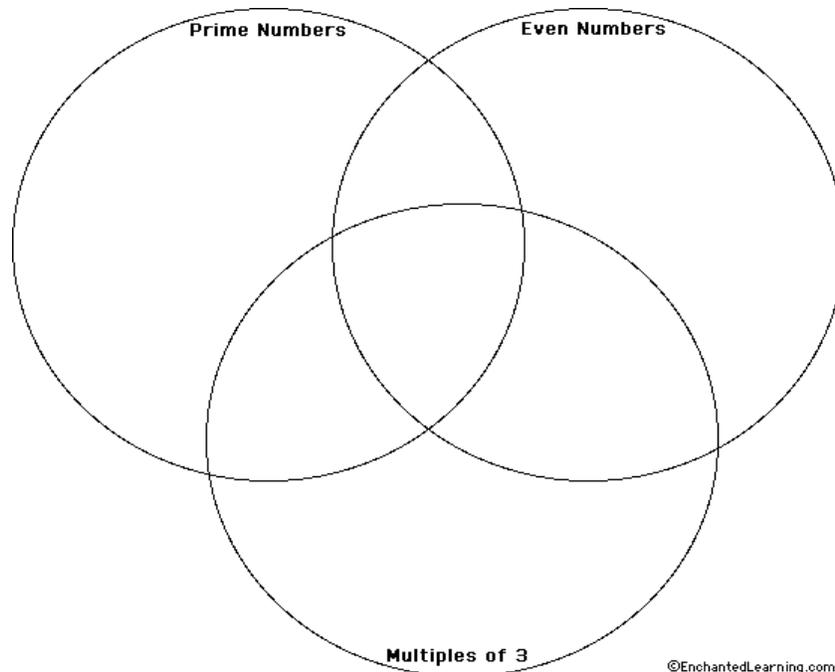
Those students outside of the circles take neither chorus nor band.

Source: <http://regentsprep.org/Regents/math/venn/Ans1.htm>

Example 3:

## Use Venn Diagrams to find the LCD (Least Common Denominator).

Task: Where does each of the following numbers belong on the Venn Diagram?  
2, 3, 4, 5, 6, 7, 8, 9, 10, 11.



Source:

<http://www.enchantedlearning.com/math/prime/venn/>

For more lesson plan using shapes, visit the following websites:

<http://illuminations.nctm.org/lessons/6-8/Sorting/SortingPolygons-AS-ShapesAll.pdf>

### STRATEGY: Using Table Charts (to Solve Word Problems)

#### Description

Graphic Organizers are concrete, pictorial ways of constructing knowledge and organize information. One type to use is the Table Chart.

#### Procedures

1. Table charts can be presented as part of the teacher's lesson. First, decide whether using a table chart is the most effective type of graphic organizer for your lesson. Also, decide when the manipulative would be most effective in your lesson: *pre-lesson* as a motivator, *during the lesson* to illustrate a point or *post-lesson* as a means of checking students' answers or as an assessment.
2. Prepare the table chart for the lesson; create appropriate headings for the columns.

3. Demonstrate to students how the table chart used is effective in helping them the lesson.
4. Provide and allow students time to utilize this tool.
5. Reinforce student's understanding of the lesson by having them create table charts.

### Application of Strategy

The teacher should model how to create table charts including creating headings for the columns, filling out the columns and arriving at a solution using the chart.

Students should then be able to create table charts independently and use it to help them arrive at a solution.

Table charts can be efficient in helping students organize new math vocabulary or concepts. They can help students organize information and arrive at solutions in Word Problems. Students can work together cooperatively creating table charts and discussing how they will be used in solving the problems.

Example 1:

Weight in ounces	Function	Price
1.0	GIVEN	\$0.42
2.0	$\$0.42 + \$0.23 =$	
3.0		
4.0		

Example 2:

### *Terms Used in Algebra*

Term	Definition	Example	Notes
binomial	algebraic expression consisting of two terms connected by + or -	$2x^2$ $a + b$	a kind of polynomial
coefficient			
constant			

## STRATEGY: Using Manipulatives to Make Word Problems Concrete

### Description

**Manipulatives** are concrete tools: visual objects that help illustrate mathematical relationships and applications. Using manipulative materials can help students to relate to real world situations which teaches them concrete understanding of the math concepts. When students develop a concrete understanding of math skills, they are more likely to perform that math skill and understand math concepts at the abstract level. Manipulatives can make math concepts come alive.

Examples of **manipulatives** include origami, paper money, rulers, dominoes, playing cards, number lines, software, buttons, blocks, cuisenaire rods, tangrams, geoboards, algebra tiles, calculators and base ten blocks.

### Procedure

1. Depending on your lesson, decide what manipulative is appropriate and most effective in demonstrating or teaching the concept of your lesson. Also, decide when the manipulative would be most effective in your lesson: pre-lesson as a motivator, during the lesson to illustrate a point or post-lesson as a means of checking students' answers or as an assessment.
2. Prepare the manipulative for the lesson
3. Show students how the manipulative is used.
4. Provide and allow students time to use the manipulative without your help.
5. Reinforce the students' understanding of the lesson by lending more examples for students to practice using the manipulative.

### Application of Strategy

Manipulatives such as playing cards can be used in teaching probability. Rulers, tangrams and geoboards can be used to teach such concepts as measurement, angles, area, geometry, decimals, factoring and estimation. Number lines, cuisenaire rods, base ten blocks and algebra tiles can be used to teach fractions, percent, prime numbers and whole numbers. Other manipulatives include graphing calculators, symbols, barculators and puzzles.

Students can work together cooperatively to solve problems and discuss mathematical ideas and concepts.

### Examples:

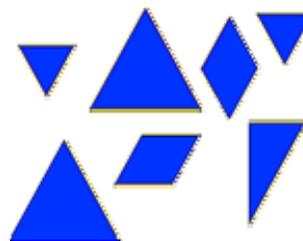
Geoboards



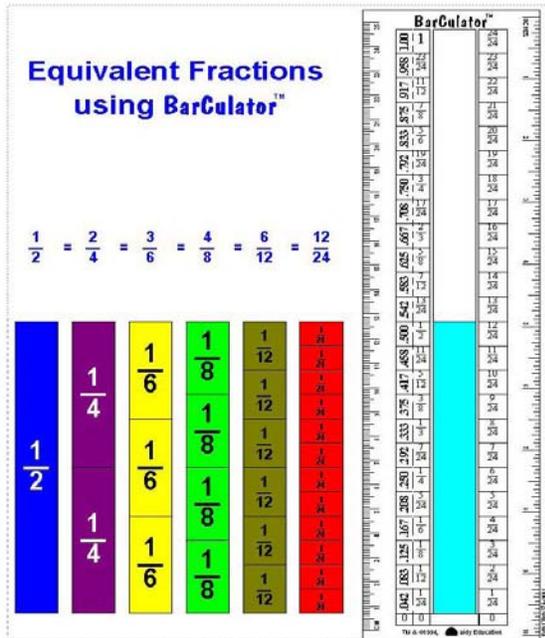
Origami



Tangrams



## Barculator



## Algebra tiles

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	x		x		
1	1	x		x		
x	x	x <sup>2</sup>		x <sup>2</sup>		
x	x	x <sup>2</sup>		x <sup>2</sup>		

### Example 2:

*To see how Archimedes calculated the value of  $\pi$  using manipulatives see following website:*

*<http://www.ima.umn.edu/~arnold/graphics.html>*

### Example 3:

For programs used to solve problems in algebra, calculus, linear algebra, and differential equations see the following websites:

<http://americanhistory.si.edu/teachingmath/html/402.htm>

## Strategies Using Students' Prior Knowledge and Culture

### STRATEGY: Cooperative Learning

Students are more likely to solve problems through discussions rather than working alone. When working in groups, each student becomes a real member of the content classroom instead of being a silent observer.

## Description

Cooperative learning groups or teams provide the ESL student with varying language and learning style experiences within the content classroom. Students learn more in classes where there is some sort of group interaction.

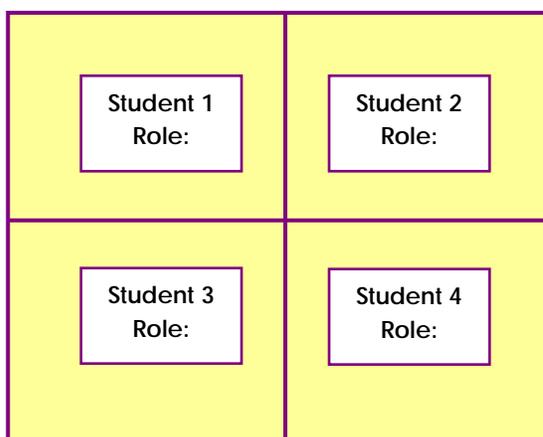
## Procedure

Teachers can pair peer partners or buddies in a variety of ways. Advanced ESL students can help those peers who are less proficient in English. Same age/grade native English speakers can be paired with non-native speakers. Second language learners can also be paired with buddies or tutors from another grade level classroom.

## Application of Strategy

Students can be assigned roles in a cooperative group in order to complete a task. For example, in solving algebraic equations, role assignments may include a manipulator, recorder, timer and a checker.

Example:



## STRATEGY: *Think Alouds*

## Description

Using a “think-aloud” technique helps students analyze and evaluate procedures by means of verbally expressing step by step how a math problem is solved. Understanding the reasons for certain procedures in math will build students’ competence and deeper understanding. In addition students will often self correct their mistakes in the process.

## Application of Strategy

Math teachers can use math think-alouds to help students make their way step-by-step to arrive at a solution. Application usage included solving word problems, logic and proofs in geometry.

1. The teacher who models the thought processes behind solving a problem may ask these questions after reading aloud the math problem:

*What am I trying to solve? What are the important \_\_\_\_\_?*

*What strategy will I use to solve the problem?*

2. The teacher models thinking aloud each step of this strategy by using helpful starting words:

first; second; the order of operations is; in order to; third; next;

I know that if I do \_\_\_\_\_, I must do \_\_\_\_\_; after this; then;

finally; does this answer make sense?

## STRATEGY: Think-Write-Pair-Share

### Description

A Think-Pair-Share is a learning strategy designed to actively involve all students in thinking about the concepts presented in the lesson rather than having the teacher pose a question and only one or two student offers a response. Think-Pair-Share also encourages student classroom participation. With Think-Pair-Share, students are given time to think through their own answers to the question before the questions are answered by other peers and the discussion moves on.

### Procedure

1. Teacher poses a discussion topic or problem to solve.
2. Give students at least 10 seconds of think time to THINK of their own answer and then write their answer.
3. Ask students to PAIR with their partner to discuss the topic or solution.
4. Finally, randomly call on a few students to SHARE their ideas with the class.

## Application of Strategy

Students can review vocabulary, new concepts, recall basic geometric terms, discuss steps or processes and solve word problems, or discuss how to rename a fraction to lowest terms.

For the ELL student, providing time to reflect and write his or her thoughts reduces anxiety.

This strategy calls for small-group interaction. All students can and will take advantage of the safe environment of small groups to use academic math language as they talk about math processes and concepts.

Example:

### THINK/PAIR/SHARE

Question: \_\_\_\_\_

Sample *Question 1: Which fraction is bigger?  $\frac{1}{3}$  or  $\frac{2}{5}$ ? How do you know?*  
*Question 2: What does factoring mean?*  
*Question 3: Do 0.2 and 0.020 equal the same fraction? Explain your answer.*

Think about your answer. Write it down if necessary.

Select a Partner.

Listen to or explain answers.

State your answer in class discussion, in written assignment or a

Switch Roles.

For more think/pair/share models in math visit this website:

<http://serc.carleton.edu/sp/merlot/math/interactive/tpshareexm.html>

### STRATEGY: Personalizing the Lesson

#### Description

Student motivation is closely connected to the positive learning environment of the classroom. Making personal connections to the concept to be taught will motivate students and connect them to the new content.

#### Application of Strategy

Prior to the start of a new concept to be taught, the teacher thinks of how to connect the student with the new content. This could be done in the form of putting

the student in a real life situation where he or she has to learn the new content in order to survive.

#### Example 1:

Pose a word problem to the students.

*Our math class is on a hike at Bear Mountain to celebrate the end of the semester. Jim and Sarah, who are hiking, decide to leave the class group and walk around a lake. They start going in the opposite directions. Jim hikes at the rate of 3 miles per hour. Sarah hikes at the rate of 2 miles per hour. The perimeter of the lake is 10 miles. How long will it be before they meet up to avoid a lecture from the teacher?*

#### Example 2:

Have students come up with sample real life situations to solve.

### Strategies for Assessing ELLs' Knowledge of Mathematics

Traditionally, the dominant mode of math assessment has been paper-and-pencil testing including matching, multiple-choice, true/false questions, problems to solve, short-answer, fill in the blank, or "show your work" questions.

English-language learners do not have to be assessed in the same way or with the same testing materials as mainstream students. Tests are not sacred documents or determiners of ESL students' abilities or any students' abilities. Students with limited English need to be graded on whether they make a sincere attempt to understand the content material at their current level of English language ability. Designing realistic assessments is not impossible to do.

### STRATEGY: Learning Logs

#### Description

Writing allows students to elaborate on their thinking and problem-solving processes and strategies. Writing about mathematics has also been shown to help articulate students' thinking as well as reinforcing their mathematical vocabulary and understanding. Equally important, writing and reflecting about math reduces math anxiety.

Keeping a learning log can be short and informal. It should *not* be graded or edited. Its purpose is for students to explore and note what they are thinking about and what they are learning.

## Learning Log Variations/Procedure

The traditional learning log asks students to summarize a class activity with the prompt:

*“What did you learn in today’s lesson?” “What questions do you still have?”*

Other Variations include:

*Life Application:* Students are asked to apply the concepts they learned in class to their life. How does the concept affect their everyday being? What would happen if they would suddenly change or cease to exist? Sample topics might be: the metric system, multiplication, the calculator and so on.

*“Showing” Vocabulary or a New Concept:* Students choose new vocabulary words to make their own. From the day’s lesson, they choose x number of words, look the word up in the dictionary and write the definition. Then they write a paragraph that shows the meaning of the word without telling the definition.

## Application of Strategy

Use learning logs to practice and strengthen new language skills and math terms in a non-threatening manner. Some ELL students may feel comfortable using their native language in their journals as a way to help solidify their understanding of math concepts.

In a learning log, students respond to a prompt that helps them articulate what they’ve learned and discover what they don’t understand. New math vocabulary can be reinforced as students reflect and write. Students can also use their learning logs as personal dictionaries.

Example:

Learning Log Prompts-

*What new concepts did you learn in this week’s lesson?*  
*What was most difficult about learning the new concept?*  
*What will help you remember or understand it?*



Students might be asked to explain a concept in their own words, such as:

*“In what instances is multiplication used in calculating probability?”*  
*“Explain the significance of the Pythagorean theorem in trigonometry.”*  
*“Explain in your own words what exponential means.”*

Students might also be asked to:

*Explain an algorithm or describe a process.*  
*Explain a theorem.*  
*Describe or interpret a graph.*  
*Discuss the solution to a problem.*  
*Write a problem or a word problem.*

## STRATEGY: Authentic Performance Tasks

### Description

Authentic assessments, such as performance tasks are activities, exercises, or problems that require students to construct solutions to questions, problems or tasks. Performance assessments contribute to student learning by giving students challenging, engaging tasks that ask them to use their knowledge. Performance tasks can provide teachers with better assessment data than traditional evaluation techniques such as tests and quizzes. Tasks should support classroom instruction.

### Procedure

1. The teacher starts with asking a student or group of students to engage in a mathematical task or investigation.
2. Examples of standard activities that create useful data about what the students know and are able to do are:
  - 1) Observations of the students as they work
  - 2) Questioning of the processes they use to solve the task
  - 3) Examination of their final results

Great care must be taken to be sure that the task is rich and motivating enough to produce sustained effort on the part of the students. Tasks which are boring or too simple will not be very good performance assessment tasks.

### Application of Strategy

Example of a Performance Task:

Title: The Career Conflict

Situation:

*Your best friend has received 2 different job offers, one in Washington and one in Oregon. Your friend comes to you for help in deciding which offer to accept. In considering the offers, keep in mind that your friend has decided to live and work in the same state. To help with the decision, you made a list of financial considerations for each job, shown in the following table.*

	Washington Job	Oregon Job
Salary	\$27,000/yr	\$30,000/yr
Federal Income Tax on Salary	25%	25%
State Income Tax on Salary	none	8%
Housing (apt.)	\$550/month	\$600/month
Food	\$175/month	\$175/month
Additional Living Expenses	\$200/month +7.8% sales tax	\$250/month +no sales tax
Car Licensing	\$350/year	\$20/year

Instructions:

1. Based on the figures in the previous table, which of the two jobs would you recommend that your friend accept? Explain, in detail, how you arrived at this conclusion.
2. How much more would your friend have to make in either Washington or Oregon to make the two jobs equally attractive from a financial viewpoint? Completely explain your answer.

Source: <http://www.esd112.org/smerc/pdf/Math-Grade9-10.pdf>

### STRATEGY: *Student Made Test Questions*

#### Description

Asking students to create test questions provides many benefits. Students must have enough mastery of the concepts to know how and what to ask. Asking students to create test questions that the teacher will then use on the actual test provides a good opportunity for review of the material learned. Students will also have a vested interest in helping design the test for the class and they will be proactive learners.

#### Procedure

1. Teacher first decides if s/he wants students to work individually or in groups.
2. Teacher tells students that they will have the opportunity to design the class test for the unit.
3. Students should look through their textbook and notes for content material to be tested.
4. Students need to write clearly the question AND the answer.
5. Teacher chooses x number of questions (ex: one question from each group) to use on the test on the following day.

#### Application of Strategy

At the end of a math unit, the teacher can ask students to write, in groups or individually, test questions for the unit.

## MINI LESSON PLAN

The following is a lesson plan that will provide information on how to teach specific content in mathematics to ELLs.

### *Using a Table Chart to Solve Word Problems*

#### Overview

While creating a chart is suggested as a means of organizing and arriving at the solution, it is up to the student to extrapolate this information and then create the headings for the columns.

#### Introduction

You have secured a summer job in a company's mailroom. You need to prove you can learn how to use the mail machine. You must weigh the mail on a scale and decide how much postage to put on each package that comes into the mailroom. Your supervisor gave you a tip suggesting it would be easier to have a chart of the prices for various weights. Currently, first class mail in the United States costs \$0.42 for the first ounce and \$0.23 for every ounce thereafter.

**Directions:** Complete this 4 step guide to help you solve the problem.

<p style="text-align: center;"><i>Understand</i></p> <ul style="list-style-type: none"> <li>• What do you know? What do you want to find out?</li> </ul>	<ul style="list-style-type: none"> <li>- first class mail in the United States costs. 42 cents for the first ounce and 23 cents for every ounce thereafter.</li> <li>- how much postage to put on each package that comes into the mailroom.</li> </ul>
<p style="text-align: center;"><i>Plan-Choose a Strategy</i></p> <ul style="list-style-type: none"> <li>• What mathematical operation will you use to arrive at a solution?</li> </ul>	<p><i>You need to combine (add) 23 cents to each previous ounce.</i></p>
<p style="text-align: center;"><i>Solve the Problem</i></p> <ul style="list-style-type: none"> <li>• Use the plan to find the solution.</li> </ul>	<p style="text-align: center;"><i>* see below</i></p>
<p style="text-align: center;"><i>Check - Look Back</i></p> <ul style="list-style-type: none"> <li>• Decide whether the answer makes sense.</li> </ul>	

*Problem Solved:*

3.0	$\$0.65 + \$0.23 =$	\$0.88
4.0	$\$0.88 + \$0.23 =$	\$1.11
5.0	$\$1.11 + \$0.23 =$	\$1.34
1.0	GIVEN	\$0.42
2.0	$\$0.42 + \$0.23 =$	\$0.65

*Sample Word Problems from the article:*

*“ESL Math and Science for High School Students:  
Two Case Studies” by George Spanos*

1. Sam's truck weighs 4,725 pounds. The truck can carry 7,500 pounds. What is the total weight of the truck and full load? (addition of whole numbers)
2. A factory that makes electronic equipment produced 3,048 VCRs in June. In July, the factory produced 2,986 VCRs and another 2,809 in August. How many VCRs were produced in the three months together? (addition of whole numbers)
3. Last week we drove to Detroit. At an average speed of 50 miles per hour, the trip took  $4\frac{1}{2}$  hours. How far was it? (multiplication of whole and mixed numbers)
4. For the first four months of the year, rain fell as follows:  $2\frac{1}{2}$  inches,  $3\frac{1}{4}$  inches,  $1\frac{1}{8}$  inches, and  $1\frac{1}{2}$  inches. What was the total rainfall for these four months? (adding mixed numbers with unlike denominators)

## RESOURCES

### Journal Articles and Essays for Teaching Mathematics to English Language Learners

<http://www.ncela.gwu.edu/pubs/reports/acadach.htm#Mathematics>

Several classroom vignettes demonstrate how problem solving in real life contexts is used to teach mathematics to ELL students.

<http://www.ncela.gwu.edu/pubs/symposia/third/spanos.htm>

This paper documents the experiences of the author in implementing a content-ESL program for high school mathematics and science. Several issues were investigated including the linguistic demands of math content and the role of learning strategy instruction.

<http://www.ncacasi.com/jsi/2002v3i1/assessment>

This article reveals several studies researching how language factors influences the validity of math assessments for ELL students.

<http://www.mathgoodies.com/articles/manipulatives.html>

This article discusses why manipulatives are valuable resources for deepening students' understanding of math including the high school level.

<http://www.uc.edu/cetl/documents/thinkalouds.pdf>

This essay describes the author's experience with invalid student learning assessments and the subsequent use of an effective technique called the Think Aloud method to evaluate deep understanding.

[http://www.coursecrafters.com/ELL-Outlook/2007/mar\\_apr/ELLOutlookITIArticle1.htm](http://www.coursecrafters.com/ELL-Outlook/2007/mar_apr/ELLOutlookITIArticle1.htm)

"Teaching Math to English Language Learners: Can Research Help?"

<http://www.ncela.gwu.edu/pubs/symposia/third/khistry.htm>

"A Naturalistic Look at Language Factors in Mathematics Teaching in Bilingual Classrooms"

<http://www.ballaghe.com/constantcontact/spring08newsletter/spring08newsletter.pdf>

"Vocabulary Development for English Language Learners"

<http://dg.icme11.org/document/get/>

This paper looks at language ability impact math achievement from the perspective of students in Latin America. However there are implications for U.S. English Language Learners.

<http://pewhispanic.org/reports/report.php?Report>

This report looks at the fast-growing number of Hispanic students designated as English language learners and how they are among the farthest behind in reading and math, according to an analysis that is based on standardized test scores.

<http://pewhispanic.org/reports/report.php?ReportID=55>

This paper examines and discusses the reasons for the high drop-out rate of foreign-born teens.

<http://www.mathematicallycorrect.com/allen4.htm>

This is a speech about language and the learning of Mathematics.

[http://www.allacademic.com/meta/p\\_mla\\_apa\\_research\\_citation/2/4/2/1/1/p242117\\_index.html#abstract](http://www.allacademic.com/meta/p_mla_apa_research_citation/2/4/2/1/1/p242117_index.html#abstract)

This paper looks at why there is no decimal metric system in the United States.

<http://www.ncela.gwu.edu/states>

This is the site of the National Clearinghouse for English Language Acquisition and Language Instruction Educational Programs.

<http://books.google.com/books?hl=en&lr=&id=BBvWHKC-2doC&oi=fnd&pg=PA17&dq=%22Cocking%22+%22Conceptual+issues+related+to+mathematics+achievement+of+...%22+&ots=Ef1sxardXM&sig=cEYdO3EIOAcczAWITiE6aiRLhc#PPA24,M1>

This paper discusses a study conducted to determine whether the language background of language minority students affect their math learning and math achievement.

<http://www.sedl.org/scimath/compass/v02n02/standard.html>

This is an excerpt from Assessment Standards for School Mathematics which examines classroom instruction as the basis for assessment.

<http://searcheric.org/digests/ed390377.html>

This is an ERIC digest article on new authentic and performance-based measures of assessment, including projects that require students to demonstrate their skills.

### Classroom Resources for the Math Teacher

<http://www.teachervision.fen.com/math/graphic-organizers/53511.html>

Ready to use graphic organizers

<http://www.numbernut.com/glossary/c.shtml>

A glossary of math terms.

[http://www.eduplace.com/state/pdf/author/chard\\_hmm05.pdf](http://www.eduplace.com/state/pdf/author/chard_hmm05.pdf)

Vocabulary strategies for Math teachers

<http://www.ima.umn.edu/~arnold/graphics.html>

Graphics for the Calculus classroom.

<http://www.uwstout.edu/soe/profdev/secondaryteamworkrubric.html>

Teamwork/Cooperative Learning Rubrics to use for self-assessment and peer feedback.

<http://www.webmath.com/>

This mathematics web site provides assistance with solving math problems.

<http://www2.ups.edu/community/tofu/lev2/journaling/writemath.htm>

A terrific site suggesting ways to use writing in the math class.

<http://www.utc.edu/Faculty/Deborah-McAllister/camta04origami.html>

A link to a slew of websites discussing and showing how to use origami in math.

<http://www.websudoku.com/>

Free puzzles with various levels of difficulty. Provides print and online syndication.

<http://www.funbrain.com/>

For teachers and students. Fun math-related activities

<http://teachingtoday.glencoe.com/lessonplans/9-12/mathematics>

Math sample lessons

<http://www.inspiration.com/Examples/Inspiration?fuseaction=math#Math>

Using Inspiration software in the Math class.

<http://www.enchantedlearning.com/graphicorganizers/math/>

Math Graphic Organizers

<http://www.teachervision.fen.com/math/teaching-methods/48952.html>

Math and Science Teaching Strategies

<http://www.nctm.org/resources/high.aspx>

National Council for Teachers of Mathematics. (Subscription required)